APPENDIX H

BASIC MATH REVIEW

A. **Numbers and Numerals**

1. *Numbers - a total quantity, or amount, of units*

2. Numerals - a word or sign, or a group of words or signs, expressing a number

B. **Kind of Numbers**

1. Whole numbers (Natural, integer) - 549

2. Fractions - subdivision of a whole number - 4/7

3. Mixed numbers - consisting of a whole number plus a fraction - 3 7/8

4. Abstract, or pure numbers (no units) - 1, 9, 10, etc. You may add, subtract, multiply or divide pure numbers

5. Concrete numbers (units) - denominate number - 4 grams, 8 ounces, 12 grains, etc. You may add or subtract one concrete number from another, but a concrete number may be multiplied or divided by a pure number.

Examples:
$10\ g + 5\ g = 15\ g$
$10\ g - 5\ g = 5\ g$
$300 \text{ grains} \times 2 = 600 \text{ grains}$
$12 \text{ ounces} \div 3 = 4 \text{ ounces}$

C. **Enumeration and Notation**

1. Enumeration - act of counting

2. Notation - recording of the number .... is the tally.

3. Denary (decimal) system of notation - record by tens

4. Base - the number 10, for example

D. **Roman Numerals** - The roman numeral system uses letters to represent numbers. It does not have a constant base system and makes calculations difficult. The system is used only to identify values.

Roman numerals	Arabic equivalent
I, i	1
V, v	5
X, x	10
L, l	50
C, c	100
D, d	500
M, m	1000
V	5000
C	100,000
D	500,000
M	1,000,000

1. When a numeral of lesser value follows one of greater value, add the two values.

Example: VI = 5 + 1 = 6
LX = 50 + 10 = 60
MDC = 1000 + 500 + 100 = 1600

2. When a number of lesser value precedes one of greater value, the value of the first is subtracted form the value of the second. The numerals V, L, and D are never used as a subtracted number.

Example: IV = 5 - 1 = 4
XL = 50 - 10 = 40
XLV = 50 - 10 + 5 = 45 (not VL)

3. When numerals of the same value are repeated in sequence, add the values.

Example: XX = 10 + 10 = 20
XXX = 10 + 10 + 10 = 30
III = 1 + 1 + 1 = 3

<u>NOTE</u>: Numerals are never repeated more than three times in sequence. For example 40 is represented as XL and not XXXX.

4. When a number of lesser value is placed between two numerals of greater value, the numeral of lesser value is subtracted from the numeral following it.

Example: XIV = 10 + 5 - 1 = 14
XXIX = 10 + 10 + 10 -1 + 29

5. Arrange the letters in order of decreasing value from left to right except for letters having values to be subtracted from the next letter.

Example: MCM = 1900
MCMXLIV = 1944

E. **Basic Arithmetic Operations** (See Basic Mathematics Homework)

1. Addition

2. Subtraction

3. Multiplication

4. Division

NOTE: Review the rules for manipulation of signed numbers (+ or -) in addition, subtraction, multiplication and division.

NOTE: Review the rules for order of operations when carrying out several types of computations in the same problem.

F. **Operations with Fractions**

1. Common fractions

a. $\frac{1}{2}$ numerator / denominator

b. Proper fraction $\frac{2}{3}$ small number (numerator)

c. Improper fraction $\frac{5}{2}$ larger number (numerator)

d. Mixed number 2 1/2

2. Adding or subtraction - lowest common denominator

Example:

X = 3/4 - 3 1/6 + 8 5/12

Solution:

X = 3/4 - 3 1/6 + 8 5/12 =

$$3/4 - 19/6 + 101/12 = \frac{9 - 38 + 101}{12} = 6$$

3. Multiplying

   Example:

   $X = 16 \times 3/4 \times 5/6$

   Solution:

   $$X = 16/1 \times 3/4 \times 5/6 = \frac{16 \times 3 \times 5}{1 \times 4 \times 6} = 10$$

4. Dividing - invert the divisor and multiply

   Example:

   $X = 6\ 1/2 \div 3/4$

   Solution:

   $$X = 6\ 1/2 \div 3/4 = 6\ 1/2 \times 4/3 = \frac{13 \times 4}{2 \times 3} = 8\ 2/3$$

5. Decimal fractions

   a. Adding and subtracting

   Example:

   1) $X = 16.34 + 176.00 + 3.41$

   $$\begin{array}{r} 16.34 \\ 176.00 \\ \underline{3.41} \\ X = 195.75 \end{array}$$

   2) $X = 1.436 - 0.471$

   $$\begin{array}{l} 1.436 \text{ minuend} \\ \underline{0.471} \text{ subtrahend} \\ X = 0.965 \text{ remainder} \end{array}$$

   b. Multiplication

   Example:

X = 7.33 x 4.7 7.33 and 4.7 are called factors

7.33 multiplicand
x4.7 multiplier
34.451 product

0.00043
x 0.2
0.000086

NOTE: The product of two decimal fractions contains as many decimal places as the sum of the decimal places in the two quantities multiplied

c. Division

Example:

$X = 258.98 \div 28.4$

258.98 = dividend

28.4 = divisor

Solution:

$$28.4 \,\overline{)\,258.980} = 9.12$$

, X = 9.12 = quotient

NOTE: The quotient involving two decimal fractions contains as many decimal places as there are places in the dividend minus the number of places the divisor. NOTE: Zeros added to the dividend must be counted as decimal places.

6. Changing common fractions to decimal fractions

Example: X = 4/5 = .8

7. Percent - parts out of 100 - convert to decimal

Percentage is a ratio indicating the number of parts out of 100. That is, 5% means 5 parts out of 100, or 5/100 (5:100). A common fraction represents a ratio, and a decimal fraction a given portion of a unit value.

a. Example: Express each of the following as percentage:

1) 11/36

Since percentage is parts per 100, then:

$11/36 = X/100$

$X = (11/36 \times 100)\%$

$X = 31\%$

2) .24

Since a decimal fraction represents a given portion of the unit value, then:

$0.24/1 = X/100$

$X = (0.24 \times 100)\%$

$X = 24\%$

G. **Mathematical Operations Using Symbols**

1. Algebraic symbols

<u>NOTE</u>: Algebra is the shorthand of mathematics.

a. <u>Letters</u> a, b, c, ... x, y, z represent certain quantities.... either known or unknown.

b. Coefficient - The number of times a single algebraic quantity is to be taken is indicated by a number before the letter.

Example: 3b, 3 = coefficient and the expressions is equivalent to b + b + b

2. Signs of algebra

a. +a, +b = quantity is positive (quantity greater than zero)

b. -a, -b = quantity is negative

3. Parentheses

a. $+(a + b) = +a + b$

b. $-(a + b) = -a - b$

c. $-(a - b) = -a + b$

H. Calculations of algebraic quantities

1. Addition and subtraction (of like factors)

a. Addition of algebraic quantities

1)
$$\begin{array}{r} +7b \\ +3b \\ \underline{+5b} \\ +15b \end{array}$$

2)
$$\begin{array}{r} -12c \\ -\ 4c \\ \underline{-\ 6c} \\ -22c \end{array}$$

NOTE: A number of like algebraic terms of like sign may be added by arranging in a column and adding together the coefficients, the sum having the same sign as the parts.

3)
$$\begin{array}{r} +9a \\ +3a \\ \underline{-4a} \\ +8a \end{array}$$

4)
$$\begin{array}{r} +\ 5b \\ +\ 4b \\ \underline{-12b} \\ -\ 3b \end{array}$$

NOTE: The negative coefficients are subtracted form positive

5) $5a + 14b + 10c$, $2b - 6c$, $3a - 9c + 3x$, and $-12b - 11c$, $-x$ are to be added

$$\begin{array}{rrrr} 5a & +\ 14b & +\ 10c & \\ & 2b & -\ \ 6c & \\ 3a & & -\ \ 9c & +\ 3x \\ \hline & -\ 12b & -\ 11c & -\ \ x \\ 8a & +\ \ 4b & -\ 16c & = 2x \text{ (answer)} \end{array}$$

b. Subtraction of algebraic quantities

NOTE: To subtract algebraic quantities, change the sign of the number to be subtracted and then combine the two numbers as in addition.

1)
$$\begin{array}{r} 15x \\ \underline{6x} \\ 9x \end{array}$$
changing the sign of 6x makes it -6x. adding 15x and -6x gives 9x.

2)
$$\begin{array}{r} -15x \\ 6x \\ -21x \end{array}$$

3)
$$\begin{array}{r} 7x -\ \ 3y \\ \underline{5x + 12y} \\ 2x - 15y \end{array}$$

c. Additional examples

1) $4x + 3x = 7x$

2) $6X^2 - 2x^2 + x = 4x^2 + x$

3) $3x^2 - 2x - 5x = 3x^2 - 7x$

NOTE: Only coefficients of like factors may be added or subtracted

2. Multiplication and division (including exponents)

a. Multiplication of simple quantities

NOTE: The parts of an algebraic expression separated by plus or minus signs are called terms. Expression containing one term is called a monomial.

1) $+a \times +b = +ab$ 2) $-a \times -b = +ab$

3) $-a \times +b = -ab$ 4) $+4b \times -c = -4bc$

5) $+6b \times +3c = +18bc$

6) $-4ax \times 5ab = -20aabx = 20a^2bx$

7) $3x \times 2x^2 = 6x^3$

b. Multiplication of compound quantities

Multiply $3x - 2y$ by $x + 4y$

Solution:

$$\begin{array}{r} 3x - 2y \\ \underline{x + 4y} \\ 3x^2 - 2xy \qquad \\ \underline{+ 12xy - 8y^2} \\ 3x^2 + 10xy - 8Y^2 \end{array}$$

FOIL Method

L

I

$(3x - 2y)\ (x + 4y)$

F

O

F = First
O = Outer
I = Inner
L = Last

$$\begin{array}{r} 3x^2 + 12xy \phantom{{}- 8y^2} \\ \underline{- \ 2xy - 8y^2} \\ 3x^2 + 10xy - 8Y^2 \end{array}$$

c. Division

1) $12ax \div 6a = \frac{12ax}{6a}$ $\frac{\text{Dividend}}{\text{divisor}}$ = 2x quotient

NOTE: In the example above express as a fraction.... common factors are canceled.

2) $\frac{9b}{3bx} = \frac{3}{x}$

3) $\frac{X^2Y^4}{X\ Y^2}$; $\frac{X \times X \times Y \times Y \times Y \times Y}{X \times Y \times Y} = XY^2$

NOTE: Since $X^2 = X \times X$ and $Y^3 = Y \times Y \times Y$, powers may be factored and the common factors canceled.

4) $\frac{12X^2 \times X^2}{4X^3} = 3X$

5) $\frac{9xy + 2y}{3xy} = \frac{9xy}{3xy} + \frac{2y}{3xy}$

$x = 2$
$y = 3$

$\frac{9(2)(3) + 2(3)}{3(2)(3)} = 3.33$

$\frac{9(2)(3)}{3(2)(3)} + \frac{2(3)}{3(2)(3)} =$

$\frac{54}{18} + \frac{6}{18} =$

$3 + .33 = 3.33$

d. Factoring

When a number is the product of two other numbers, the component parts are known as factors.

Example:

$12a^3x^2 + 33a^2x^2 - 18ax^3 + 9ax$

Common factor = 3ax

therefore: $3ax(4a^2x + 11ax - 6x^2 +3)$

Example:

$3x^2 + 9bx + 24xy + 4ax + 12ab + 32ay$

$3x(x + 3b +8y) + 4a(x + 3b + 8y)$

$(3x + 4a)\ (x + 3b + 8y)$

e. Powers and exponents

<u>NOTE</u>: 4 is the second, and 8 is the third power of 2.

1) $AX \times AX \times AX \times AX = A^4X^4$; AX is the root and A^4X^4 is called the power, and 4 = exponent

2) $+A \times +A = A^2$ 3) $-A \times -A = +A^2$
$(-A)^2 = +A^2$

4) $(-A)^3 = -A^3$
$(-A)^4 = +A^4$
$(-A)^5 = -A^5$

5) $X^2 \times X^3 = X^5$ 6) $X^2Y \times XY = X^3Y^2$

7) $4XY(-3XZ) = -12X^2YZ$ 8) $X^5 \div X^3 = X^{5-3} = X^2$

9) $X^2 \div X^3 = X^{2-3} = X^{-1}$ 10) $\frac{2ab}{X^3} = 2abx^{-3}$

f. Solve for one unknown, x

Example:

36X = 756 + 8X

Solution:

36X - 8X = 756

28X = 756

X = 27

I . **Ratio and Proportion**

1. General form: a:b = c:d; a:b::c:d; a/b = c/d

   Expression is read a is to b as c is to d

   Product of means = product of extremes

2. Extremes (Outer members)

   a, d = extremes ("outer members")

3. Means (Middle members)

   b,c = means ("middle members")

NOTE: If a/b = c/d, then a = bc/d, b = ad/c, c = ad/b, d = bc/a

4. Solution (product of means = product of extremes)

   Example:

   a. Solve for x

$$\frac{122/}{11.2} = \frac{24}{x},\ x = \frac{(11.2)(24)}{(122)},\ x = 2.2$$

b. If 3 tablets contain 15 grains of aspirin, how many grains should be contained in 12 tablets.

$$\frac{3}{12} = \frac{15}{x},\ x = \frac{12 \times 15}{3} = 60 \text{ grains}$$

J. **World Standards for Units of Measurement**

1. Centimeter - gram - second (cgs) = metric system
2. Foot - pound - second (fps) = English system

K. **Advantages of the Metric System**

1. Simple - conversion based on powers of ten
2. Reproducible - based on non-changeable physical constants
3. Interrelated - One unit based on another

L. **Metric Prefixes**

1. Kilo - 1000 - 10^3
2. Hecto - 100 - 10^2
3. Deca - 10 - 10^1
4. (Unit) - 1 - 10^0
5. Deci - 0.10 - 10^{-1}
6. Centi - 0.01 - 10^{-2}
7. Milli - 0.001 - 10^{-3}
8. Micro - 0.000001 - 10^{-6}
9. Nano - 0.000000001 - 10^{-9}
10. Pico - 0.000000000001 - 10^{-12}

M. **Metric Units**

1. Length

   a. Basic unit: meter

NOTE: Use above prefixes in interconversion.

b. Definition: The International System of Units (SI) has defined the meter as 1.65 x 10^6 wavelengths of the orange-red light emitted from the element Krypton 86. The SI now defines the standard meter as how far light will travel in 1/299,792,458 of a second. It is thus based on the speed of light as measured by an "atomic clock."

c. Other lengths: Micron (micrometer) 10^{-6} m

Angstrom (A) 10^{-10} m

2. Volume

a. Basic unit: liter

b. Definition: Volume occupied by one kilogram of water at four degrees centigrade. The SI unit for volume is the cubic meter (m^3).

c. Other units: Lambda (λ) = microliter (uL) = 10^{-6} L

3. Mass

a. Basic unit: gram

b. Definition: One-thousandth of the weight of the international prototype kilogram (1889 - a block of platinum-iridium alloy stored under noncorrosive conditions by the International Bureau of Weights and Measures in a vault in Sevres France).

4. Prefixes combined with basic units

a. Prefix first, unit second

b. Sample interconversions

N. **Metric - English conversions**

<u>NOTE</u>: Use the factor-label method in calculations

Prefix	Abbreviation	Scientific Notation	Numerical Value
Mega-	M	10^6	1,000,000
Kilo-	k	10^3	1,000
Hecto-	h	10^2	100
Deka	da	10^1	10
-----	--	10^0	1
Deci-	d	10^{-1}	0.1
Centi-	c	10^{-2}	0.01
Milli-	m	10^{-3}	0.001
Micro-	u	10^{-6}	0.000001
Nano-	n	10^{-9}	0.000000001

Constants Relating Units

A		C		
cm	x	10^8	=	Angstroms
cm	x	10^7	=	Millimicrons
cm	x	10^4	=	Microns
Cubic cm	x	2.64×10^{-4}	=	Gallons
Cubic in.	x	16.4	=	Cubic centimeters
ft.	x	30.5	=	Centimeters
in.	x	2.54	=	Centimeters
kg	x	2.20	=	Pounds
km	x	0.621	=	Miles
L	x	0.264	=	Gallons
L	x	61.0	=	Cubic inches
L	x	1.06	=	Quarts
m	x	1.09	=	Yards
m	x	3.28	=	Feet
m	x	39.4	=	Inches
oz.	x	28.3	=	Grams
pd.	x	454	=	Grams
qt.	x	946	=	Cubic centimeters
sq. in.	x	6.45	=	Square centimeters

O. **Temperature Scales**

1. Fahrenheit (°F)

   a. 0°F = freezing point of sea water
   b. 32°F = freezing point of pure water
   c. 98.6°F = body temperature
   d. 212°F = boiling point of pure water
   e. The temperature interval on the Fahrenheit scale, between the boiling and freezing point, is 180 °F.

2. Centigrade (°C = t)

   a. 0°C = freezing point of pure water

   b. 100°C = boiling point of water

   c. 37°C = body temperature

   d. Absolute zero = 1273 °C

   e. The temperature interval on the Centigrade scale is 100 °C3.

3. Kelvin (Absolute, °K = T)

   a. Absolute zero (0 °K): temperature where are molecular motion stops

   b. 0 °K = -273 °C

   c. 0 °K = -459 °F

   $T = 273 + t$

   $t = T - 273$

4. Interconversion of Fahrenheit and Centigrade temperatures

NOTE: An interval of 180 °F = 100 °C
An Interval of 1 °F = 5/9 °C = 1/1.8 °C
An interval of 1 °C = 9/5 °F = 1.8 °F

   a. Formula: °F = (1.8 x °C) + 32

   b. Formula: °C = (°F - 32)/1.8

c. Examples

1) 37 °C ______________ °F

°F = (1.8 x °C) + 32
°F = (1.8 x 37) + 32
°F = 66.6 + 32
°F = 98.6

2) 75 °F __________________ °C

°C = (°F - 32)/1.8
°C = (75 - 32)/1.8
°C = 43/1.8
°C = 23.9 °C

P. **Examples and Applications of Principles of Significant Digits**

1. Numbers of significant digits

2.1 had two significant digits

2.33 has three

0.000002 has one
zero is positional
1,000,000 has one

1,000,001 has seven - zeros are significant

0.21 has two

0.20 has two

2. Significant digits and round-off in mathematical operations

	Given	Round
Addition:	12.7	12.7
(Subtraction)	13.62	13.6
	0.754	0.8
		27.1

Multiplication and division

2.2 x .55 = 1.210 -----> 1.2

1.5 ÷ .5 = 3

6.751 x .2 = 1.3502 ----> 2

Q. Examples of Operations with Fractions

1. Common fractions:

$$3/4 - 3\ 1/6 + 2\tfrac{1}{2} = 3/4 - 19/6 + 5/2 = 9/12 - 38/12 + 30/12 = \frac{9 - 38 + 30}{12} = 1/12$$

$$3/2 \times 1/5 = \frac{3 \times 1}{2 \times 5} = 3/10$$

$$4/5 \div 1/6 = 4/5 \times 6/1 = \frac{4 \times 6}{5 \times 1} = 24/5 = 4\ 4/5$$

2. Decimal Fractions:

$$\begin{array}{r} 1.2 \\ 2.3 \\ \underline{+4.2} \\ 7.7 \end{array} \qquad \begin{array}{r} 0.00043 \\ \underline{\times\ \ 0.2} \\ 0.000086 \end{array} \rightarrow 0.9 \qquad \begin{array}{r} 0.00043 \\ \underline{\times\ \ 0.20} \\ 0.0000860 \end{array} \rightarrow 0.000086$$

3. Conversion (by division)

$$4/5 \qquad \begin{array}{r|l} & \ \ .8 \\ \hline 5 & 4.0 \\ & 4\ 0 \end{array}$$

4. Percent

<u>NOTE</u>: $\% = \frac{\text{part}}{\text{total}} \times 100$; $\text{Part} = \frac{(\%)(\text{total})}{100}$

Example: A college has 4520 male and 3327 female students enrolled. What percent of the student body is female?

Solution: $\% = \frac{\text{part}}{\text{total}} \times 100$; $\%\text{female} = \frac{3327}{7847} \times 100 = 42.39$

The human body is approximately 70 percent water by weight. What is the weight of water in a 170 pound person?

$$\text{Part} = \frac{(\%)(\text{total})}{100} \qquad \text{Weight of water} = \frac{(70)(170)}{100} = 119 \text{ lb}$$

R. **Examples of Exponential Numbers**

Base 2

$2^2 = 2 \times 2 = 4$

$2^4 = 2 \times 2 \times 2 \times 2 = 16$

Base 10

$10^2 = 100$

$10^4 = 10{,}000$ exponent "locates" decimal point

$10^{-4} = 0.0001$

Multiplication

$(2.2 \times 10^5) \times (2.2 \times 10^2) = 4.84 \times 10^7 \text{ ---> } 4.8 \times 10^7$

$(2.2 \times 10^5) \times (2.2 \times 10^{-2}) = 4.8 \times 10^3$

Division

$$(2.2 \times 10^5) \div (2.0 \times 10^2) = \frac{2.2 \times 10^5}{2.2 \times 10^2} = 1.1 \times 10^3$$

$$(2.2 \times 10^5) \div (2.0 \times 10^{-2}) = \frac{2.2 \times 10^5}{2.0 \times 10^{-2}} = 1.1 \times 10^7$$

S. **Examples of Logarithmic Operations**

1. $y = a^x$

Y is a number, a is the "base", x is the power to which the base must be raised to produce the number y. This exponent is the logarithm. When y is 8 and a is 2, then x = 3.

$2 \times 2 \times 2 = 8$

Base 10 and "e" are commonly used. We are concerned only with base 10. Operations are similar with both.

$1000 = 10^{3.000}$ $\log_{10}1000 = 3$ $\text{antilog}_{10}3 = 1000$

2. $0.001 = 10^{-3.000}$

Characteristic: whole number which locates the decimal point

Mantissa: decimal fraction which determines the number.

log (2.2×10^4) = log 22000 = log 2.2 + log 10^4
(follows exponential rules of multiplication and division)

log 10^4 = 4, log 2.2 = ? Go to table for mantissa:

log 2.2 = .3424, so log 22000 = 4.3424

3. log 6.35 = log (6.35×10^0) = 0.80277

log 635 = log (6.35×10^2) = 2.80277

log .635 = log (6.35×10^{-1}) = $\bar{1}.80277$

log .0635 = log (6.35×10^{-2}) = 2.80277

antilog 0.xxxx = y.yyy

antilog 2.xxxx = yyy.y

antilog $\bar{2}$.xxxx = .0yyyy

antilog $\bar{1}.80277$ = 6.35×10^{-1} = .635

4. Multiplication and division by converting to logs, adding to multiply and subtracting to divide:

450 x 50.0 or log 450 + log 50.0

= 2.6532 + 1.6990 = 4.3522

antilog 4.3522 = 2.25×10^4 = 22,500

Similarly: 450 ÷ 50.0 or log 450 - log 50.0

= 2.6532 - 1.6990 = .9542

antilog .9542 = 9.00

T. Illustrative Examples of Mathematical Operations Using Symbols

1. *Addition and Subtraction*

$\begin{array}{r} 3x \\ \underline{+2x} \\ 5x \end{array}$	3 and 2 are coefficients x is an unknown factor

$3x + 2y = 3x + 2y$ Let x = apple
y = orange

Let x = square feet of floor
y = cubic yards of concrete

Let x = dose in mg/kg
y = flow rate

2. Multiplication and Division

$3x \times 2y = 6xy = (6)(x)(y)$

When $x = 2$ and $y = 3$;

$(3)(2) \times (2)(3) = 36$ $\quad 6 \times 2 \times 3 = 36$

$$\frac{6}{6x} = \frac{1}{x}$$

3. *Linear equations:*

$y = a + bx$ a and b are coefficients
Either x or y must be known to calculate value of other i.e., there can be only one "unknown" in an equation or formula

Time left equals pressure times a tank constant divided by the flow rate or:

$$t = \frac{pc}{f}$$

If the pressure on an oxygen tank is 6 psi, the tank constant is 3 and the flow rate is 2 liters/min, how much longer will the oxygen in the tank last?

$$t = \frac{pc}{f} = \frac{6 \times 3}{2} = \frac{18}{2} = 9$$

If a tank whose pressure starts at 6 psi can provide 9 minutes of oxygen at a flow rate of 2 L/min., what is the tank constant?

$$t = \frac{pc}{f} \qquad ft = pc \qquad c = \frac{ft}{p}$$

$$= \frac{2 \times 9}{6} = 3$$

U. Ratio and Proportion Examples

Symbolic: let a, b, c, and d represent numerals

$a:b = c:d$

$a:b :: c:d$ "a is to b as c is to d"

$a/b = c/d$

Product of the means = product of the extremes

$a/b = c/d \quad ad = cb$

Problem solving:

$1/2 = 2/4 \qquad 1 \times 4 = 2 \times 2 = 4$

If 1 tablet contains 20 grains of aspirin, how many grains should be contained in 2 tablets?

$1/20 = 2/? \quad 1 \times ? = 2 \times 20$

$? = 40$

A medication contains 20 mg of active drug per cc and the suggested dosage is 0.4 mg/kg. What volume is required for a 120 kg patient?

0.4/1 = ?/120; (.4)(120) = (?)(1); ? = 48 mg required

20/1 = 48/?; 48 = 20?; ? = 48/20 = <u>2.4 cc</u>

Percent: 5% = 5 out of 100

11/16 = ?/100; ? = 11/36 x 100 = 31%

.24/1 = ?/100; ? = 100 x .24 = 24%

V. Basic Math Homework

NOTE: If available, perform these calculations on your calculator. The purpose of the following exercises is to get practice on your calculator.

1. Add the following:	Answers:
a. 32 + 27 =	59
b. 6.35 + 2.47 =	8.82
c. 3.03 + .6 =	3.63
d. 7 + 2.45 =	9.45

2. Find the following differences:

   a. $347 - 281 =$ 66

   b. $6.27 - 2.04 =$ 4.23

   c. $5 - 2.11 =$ 2.89

   d. $(-3) - 7 =$ -10

   e. $12 - 18 =$ -6

3. Find the following products:

   a. $5 \times 18 =$ 90

   b. $2.4 \times 6 =$ 14.4

   c. $3.12 \times 2.5 =$ 7.8

   d. $(-2.3)(-3.5) =$ 8.05

   e. $(-3)(7) =$ -21

4. Find the following quotients:

   a. $52 \div 2 =$ 26

   b. $26 \div 14 =$ 1.86 or 1 6/7

   c. $35 \div 0.7 =$ 50

   d. $(-9) \div 3 =$ -3

   e. $34.2 \div .6 =$ 57

5. Perform the following calculations:

   a. $2 + 3 + 4 - 6 =$ 3

   b. $(3 - 6)\ (4 + 2) =$ -18

   c. $8/3 + 1/6 - 5/2 =$ 1/3

   d. $\dfrac{3 \times 10^2}{2 \times 10^4} =$ 1.5×10^{-2}

   e. $(4 \times 10^{-2})\ (3 \times 10^4) =$ 1.2×10^3

   f. $\dfrac{6 \times 10^{-2}}{4 \times 10^{-4}} =$ 1.5×10^2

6. Convert the following percents to decimals, and decimals to percent:

a. 3% =	0.03
b. 2.5% =	0.025
c. 45% =	0.45
d. 87.5% =	0.875
e. 0.62 =	62%
f. 3.225 =	322.5%
g. 0.025 =	2.5%

7. Find the following:

log 10,000 =	4.0000
log 10^{-4} =	$\bar{4}.0000$
antilog 3.0000 =	1000
antilog 2.4487 =	281
log 0.0003 =	$\bar{4}.4771$
log 0.001 =	$\bar{3}.0000$
antilog $\bar{4}.0000$ =	0.0001
antilog $\bar{3}.6675$ =	0.00465

8. Perform the following calculations:

a. $3X + 7 = 4X - 1$	X = 8
b. $\frac{7X - 3}{5} = 1 - X$	X = 0.666
c. $2(X + 1) - 2 = 3(x - 3) + 9$	X = 0
d. $5X - 3(1 - X) = 2X + 21$	X = 4
e. $(6)(1.0) + (6)(0.21) = (0.605)(6 + X)$	X = 6

f. $\dfrac{X}{273 + 17} = \dfrac{2{,}150}{273 + 22}$ X = 2113.56

g. $\dfrac{2{,}100}{332 + 23} = \dfrac{X}{432 + 31}$ X = 2738.87

9. Perform the following conversions:

   a. 0.0132 ml = 13.2 ul

   b. 502 mg = 0.502 g

   c. 37.5 cm = 0.000375 km

   d. 200 mL = 2.00 dL

   e. 30 dL = 3000 mL

   f. 16.5 km = 1,650,000 cm

10. How many:

a. mL are there in 300 dL?	30,000 mL
b. kg are there in 500 mg?	0.000500 kg
c. mg are there in 0.20 g?	200 mg
d. L are there in 150 mL?	0.150 L
e. cm are there in 2.55 m?	255 cm
f. g are there in 275 mg?	.275 g

11. Given: 2.54 cm/in; 2.20 pounds/kg; 28.3 g/oz

    How many:

a. oz are there in 75 g?	2.65 oz
b. cm are there in 2 ft?	60.96 cm
c. kg are there in 2 pounds?	0.909 kg
d. in. are there in 2 m?	78.8 in
e. g are there in 16 oz?	452.8 g
f. ft are there in 1 meter?	3.28 ft

MATHEMATICS EXAMINATION

NOTE: Work each of the following math problems. Check your answer against the answers provided.

1. Express each of the following as the product of two numbers, one of which is an integer power of ten.

   a. 22,400 — Ans. 2.24×10^4

   b. 364 — Ans. 3.64×10^2

   c. 0.364 — Ans. 3.64×10^{-1}

   d. 0.000346 — Ans. 3.64×10^{-4}

2. Evaluate x in each of the following as a power of ten.

   a. $x = 22.400 \times 250$ — Ans. 5.6×10^6

   b. $x = 72{,}000 \div 340$ — Ans. 2.13×10^2

   c. $x = 340 \div 72{,}400$ — Ans. 4.70×10^{-3}

3. Solve for x

   a. $35x = 450 + 5x$ — Ans. 15

   b. $x = 36 \times \frac{18}{5 \times 18} \times \frac{24 + 6}{6}$ — Ans. 36

4. Determine the equality for the following algebraic equations

   b. $4x + 3x =$ — Ans. $7x$

   c. $6x^2 - 2x^2 + x =$ — Ans. $4x^2 + x$

   d. $3x^2 - 2x - 5x =$ — Ans. $3x^2 - 7x$

   e. $3x \times 2x^2 =$ — Ans. $6x^3$

   f. $12x^2 \div 3x =$ — Ans. $4x$

   g. $\frac{12x^2 \times x^2}{4x^3} =$ — Ans. $3x$

5. Solve for x:

   a. $\frac{12.2}{x} = \frac{4.8}{33.4}$ — Ans. 84.9

b. A sample of brass weighing 4.55 lb. was found to contain 3.18 lb. of copper and 1.37 lb. of zinc. How much copper (x) would there be in 500 lb. of brass?

<u>NOTE</u>: Use ratio and proportion.

Ans. 349 lb. copper

c. Express each of the following as a percentage:

<u>NOTE</u>: Use ratio and proportion.

1) 11/36 — Ans. 31%

2) 0.24 — Ans. 24%

<u>NOTE</u>: Use ratio and proportion.

d. Given that 140 ml. of chlorine gas weighs 0.450 g.; find the density of chlorine in grams per liter (g/L).

<u>NOTE</u>: Remember that 1 L. = 1000 mL.

Ans. 3.21 g./L

6. Perform the following metric conversions

a. 125 mm to cm. — Ans. 12.5 cm.

b. 2600 m to km. — Ans. 2.6 km.

c. 25 in. to cm. — Ans. 63.5 cm.

d. 18 ft. to m. — Ans. 5.49 m.

e. 0.025 L to mL. — Ans. 25 mL.

f. 2.35 qt. to mL. — Ans. 2223 mL.

g. 1.65 kg. to g. — Ans. 1650 g.

h. 35 lb. to kg. — Ans 15.9 kg.

i. How many $cm.^3$ are there in a metal bar of uniform cross section measuring 10 cm. by 2 cm. by 90 cm.?

Ans 1800 $cm.^3$

j. Calculate the volume in liters of a cube measuring 12 cm. by 120 mm. by 1.2 m.

Ans. 17.3 L.

k. A uniform iron bar 15 in. long weighs 2 lb. 4 oz. Calculate the weight of the bar in grams per centimeter of length.

Ans. 26.8 g.

7. Solve the following equations.

a. $^{\circ}F = 9/5(50^{\circ}C) + 32$ Ans. $122^{\circ}F$

b. $^{\circ}C = 5/9(50 - 32)$ Ans. $10^{\circ}C$

8. Using significant digits perform the following mathematical operations.

a. 2.1 m. + 0.3 m. + 2.07 m. + 3.224 m.

Ans. 7.7 m.

b. 21.630 L. + 844 mL. + 0.036 L. + 10.196 mL.

Ans. 22.520 L.

c. 13.22 cm. - 28.36 mm. Ans. 10.38 cm.

d. 32.57 X 7.14 Ans. 233.

e. 0.0482 X 0.2134 Ans 1.03×10^{-2}

f. $7632 \div 173$ Ans. 44.1

g. There are 2.54 cm. in one inch. How many centimeters are there in 15.0 in.?

Ans. 38.1 cm.

h. What is the circumference of a circle of 10.00 in. diameter, given that $\pi = 3.14$?

Ans. 31.4 in.

9. Perform the following calculations.

a. A pressure of 450 lb./in.^{2} was recorded on a pressure gauge of an oxygen tank. What would this be in terms of atmospheres?

NOTE: Remember that 1 atmosphere = 14.7 lb./in.^{2}

Ans. 30.6 atm.

b. A barometer registered 520 mm. of Hg on the top of Pikes Peak, Colo. This would be equivalent to how many atmospheres?

NOTE: Remember that 1 atmosphere = 760 mm. Hg.

Ans. 0.684 atm.

c. A volume of 250 ml. (V_1) of oxygen was collected at 293°K. (T_1) and 785 mm. (P_1) of Hg. The next day the temperature was 310°K. (T_2) and the pressure 770 mm. (P_2) of Hg. Calculate the resultant volume (V_2) of the oxygen.

$$\frac{(P_1)(V_1)}{(T_1)} = \frac{(P_2)(V_2)}{(T_2)}$$

V_2 = ______________? Ans. 270 ml.

10. Perform the indicated operations using logarithms.

a. 782 X 74 Ans. 57870

b. 7340 ÷ 29 Ans. 253

c. 0.0756 X 0.34 Ans. 0.0257

d. 0.314 ÷ 2.14 Ans. 0.1467

11. Perform the indicated operations using logarithms.

a. $X = \log \frac{1}{1 \times 10^{-3}}$ Ans. 3

b. $X = -\log (1 \times 10^{-3})$ Ans. 3

c. $X = -\log (0.001)$ Ans. 3

d. $X = 6.1 + \log \frac{25}{(.03)(40)}$ Ans. 7.4181

e. $X = 6.1 + \log \frac{5}{(.03)(12)}$ Ans. 7.2430

<u>NOTE</u>: Use dimensional analysis were appropriate.

12. Calculate the number of grams of sodium hydroxide that must be added to 1.00 L of water (H_2O) to make a 15.0 % (w/w) solution.

Since 1.00 L of water is equal to 1000 g of water, the problem says

1000 g H_2O = ? g NaOH (in a 15.0% solution)

In a 15.0 % solution there will be 15.0 g of NaOH for each 85.0 g of H_2O (to give a total of 100 g of solution). This gives the relationship

15.0 g NaOH = 85.0 g H_2O

which can be used as a conversion factor. Therefore, the calculation is done as

$$1000 \text{ g } H_2O \times \frac{15.0 \text{ g NaOH}}{85.0 \text{ g } H_2O} = 176 \text{ g NaOH}$$

13. (a) Derive the dimensional formula for the conversion of mi./hr. to ft./min. (b) Determine the numerical value of the constant relating the two dimensions. (c) Change 25 mi./hr. to ft./min.

Solution:

a. $\frac{\text{mi.}}{\text{hr.}} \times \frac{\text{ft.}}{\text{mi.}} \times \frac{\text{hr.}}{\text{min.}} = \frac{\text{ft.}}{\text{min.}}$

b. Assign unit value to <u>mi.</u>. Then:

$$1.00 \frac{\text{mi.}}{\text{hr.}} = 1.00 \frac{\text{mi.}}{\text{hr.}} \times 5280 \frac{\text{ft.}}{\text{mi.}} \times \frac{1}{60} \frac{\text{hr.}}{\text{min.}} = 88 \frac{\text{ft.}}{\text{min.}}$$

c. Since $1.00 \frac{\text{mi.}}{\text{hr.}} = 88 \frac{\text{ft.}}{\text{min.}}$, then:

$$25 \frac{\text{mi.}}{\text{hr.}} = (25 \times 88) \frac{\text{ft.}}{\text{min.}} = 2200 \frac{\text{ft.}}{\text{min.}}$$

or d. $\frac{ft.}{mi.} = \frac{5280\ ft.}{1\ mi.} \times \frac{25\ mi.}{1\ hr.} \times \frac{1\ hr.}{60\ min.} = 2200\ \frac{ft.}{min.}$

14. Calculate the moles of base/Liter of solution (mol/L) for a solution that contains 15 g of potassium hydroxide in 225 mL of solution.

NOTE: Moles = amount in grams, and Liter = a unit volume

There are $\frac{56\ g\ KOH}{1\ mol\ KOH}$ and $\frac{1\ L\ soln}{1000\ mL\ soln}$

Derive the dimensional formula for the conversion of g/mL to mol/L

$$\frac{g}{mL} \times \frac{mol}{g} \times \frac{mL}{L} = \frac{mol}{L}$$

$$\frac{15\ g\ KOH}{225\ mL\ soln} \times \frac{1\ mol\ KOH}{56\ g\ KOH} \times \frac{1000\ mL\ soln}{1\ L\ soln} = 1.2\ \frac{mol\ KOH}{L\ soln}$$

15. Evaluate the following numerically and dimensionally:

a. How many milliliters are there in 3.25 l.?

Ans. 3.25×10^3 ml.

b. How many feet are there in 1000 mm.?

Ans. 3.28 ft.

c. How many liters are there in 5.0 gallons of gasoline?

Ans. 18.9 L

d. $18\ ft^2 \div 3\ ft$

Ans. 6 ft.

e. 12.0 cm. X 15.0 cm X 75.0 mm

Ans. $1.35 \times 10^3\ cm^3$

f. Given that there are 30.5 cm/ft., determine the number of feet per centimeter (ft./cm.).

Ans. 3.28×10^{-2} ft./cm.

g. Convert 1.56 kilograms to milligrams

Ans. 1.56×10^6 mg.

h. In each of the following derive the dimensional formula for the conversion of the given dimensions to the desired dimensions; then determine the value of the constant relating the two dimensions.

1) mm. to km. — Ans. 10^{-6}

2) lb. to mg. — Ans. 4.54×10^6

3) $\frac{\text{ft.}}{\text{sec.}}$ to $\frac{\text{mi.}}{\text{hr.}}$ — Ans. 0.682

4) ft^3 to gal. — Ans 7.48

Remember: $231 \frac{in^3}{gal.}$

Solution:

$$\frac{\text{gal.}}{231\text{in.}^3} \times \left(\frac{12\text{ in.}}{\text{ft.}}\right)^3$$

$$\frac{\text{gal.}}{231\text{ in.}^3} \times \frac{1728\text{ in.}^3}{ft^3} = \frac{7.48\text{ gal.}}{ft^3}$$